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L1549-GB

03JUL00 E549727-1 D10017
P01/7700 0.00-0016259.4

2. Patent application number
(The Patent Office will fill in this part)

0016259.4

03 JUL 2000

3. Full name, address and postcode of the or of each applicant *(underline all surnames)*

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Interleuvenlaan 68
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B-3001 Leuven
Belgium

Patents ADP number *(if you know it)*

7913327 8001

If the applicant is a corporate body, give the country/state of its incorporation

BE

4. Title of the invention

A COMPUTER-AIDED ENGINEERING METHOD FOR PREDICTING THE ACOUSTIC SIGNATURE OF VIBRATING STRUCTURES USING DISCRETE MODE

5. Name of your agent *(if you have one)*

William E. Bird of Bird Goen & Co

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Patents ADP number *(if you know it)*

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Country	Priority application number *(if you know it)*	Date of filing *(day / month / year)*

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11.

I/We request the grant of a patent on the basis of this application.

Signature William E. Bird Date

William E. Bird 3 July 2000

12. Name and daytime telephone number of person to contact in the United Kingdom 0181-301-1129 JANE BIRD

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A Computer-Aided Engineering method for predicting the acoustic signature of vibrating structures using discrete models

DESCRIPTION OF THE INVENTION

Definitions

Acoustic radiation prediction: is an engineering process that aims at predicting the sound pressure level in the surrounding field of a vibrating structure (see figure 1). Surface vibrations create acoustic pressure waves that propagate to the receiver.

Inverse numerical acoustics: is an engineering process that aims at deriving the surface vibration velocity from field sound pressure measurements (see figure 2). As such, it is the inverse problem than acoustic radiation prediction.

Discretization Methods: in order to solve acoustic radiation prediction problems and inverse numerical acoustics problems, computer models are built using numerical methods. The most frequently used numerical methods are the boundary element method (BEM) and the finite-infinite element method (FEM/IFEM). These methods are relying upon a discretization of the geometrical domain (FEM/IFEM) or of its boundary (BEM), and are solving numerically the wave equation, in the frequency domain. There exist substantial published literature concerning both theoretical and practical aspects of these numerical methods. The major computational effort involved within the application of such numerical methods concerns the solution of a large system of linear equations.

Background of the invention

The acoustic performance of manufactured products is becoming an extremely important aspect in the design and development process, not only to improve the comfort of the user of the product (e.g. passengers in a car, in an aircraft), but also to reduce the nuisance to the surroundings (e.g. habitations close to highways, to an airport).

The invention is related to two computer-aided engineering processes that are key for evaluating and optimizing the acoustic performance of structures: the ability to predict the acoustic radiation pattern of a vibrating structure, either from computed or measured surface vibrations (acoustic radiation prediction), and the ability to recover surface vibrations onto a vibrating structure from measured field sound pressure level. The latter one is sometimes the only manner to perform source identification, when it is impossible to apply measurement devices onto the structure surface (e.g. rotating tire).

Those engineering processes are relying on numerical methods, amongst which the most popular are currently the various forms of the boundary element method (the direct mono-domain, the direct multi-domain and the indirect approaches), and the finite element method, that can be extended to handle unbounded regions, e.g. using infinite elements.

The computational cost associated with numerical methods for acoustic performance prediction is usually very large, and is linearly proportional to:

- the number of operational conditions (may be about a 100 different cases);
- the number of frequency lines to be evaluated for obtaining a representative response function (will typically be about 100 to 200);
- the number of design variants to be studied.

Acoustic Transfer Vectors

The invention relies on any of those numerical methods, and is based on the known concept of Acoustic Transfer Vectors (ATV's). ATV's are input-output relations between the normal structural velocity of the vibrating surface and the sound pressure level at a specific field point (see figure 3).. ATV's can be interpreted as an ensemble of Acoustic Transfer Functions from the surface nodes to a single field point. Literature refers sometimes to that concept as Contribution Vectors or Acoustic Sensitivities.

ATV's only depend on the configuration of the acoustic domain, i.e. the shape of the vibrating body and the fluid properties controlling the sound propagation (speed of sound and density), the acoustic surface treatment, the frequency, and the field point. They do not depend on the loading condition. The concept of ATV's, and their properties, is not new, and has already been published in several scientific papers.

Summary of the invention

The present invention will mainly be described with reference to acoustic waves but the invention is not limited thereto. The concepts underlying the present invention may be applied to any vibrating energy wave which can be described by wave equations. For example, the present invention may be applied to electromagnetic waves. The present invention may provide a method for computing a vibrating wave Transfer Vector based on the reciprocity principle, comprising the steps of:

- simulating positioning of a monopole, omnidirectional wave energy source at a reference position remote from a body;
- computing a boundary vibration amplitude of the wave generated by the source at a surface of the body;
- deriving from the boundary vibration amplitude said vibrational Transfer Vector.

The present invention may also include a method for computing an additional vibrational Transfer Vector comprising the steps of:
computing at least a first and a second vibrational transfer vector at a first and a second predetermined frequency, respectively, and

computing the additional Transfer vector at a frequency intermediate the first and second frequency by interpolation between the first and second tranfer vectors.

The present invention may also include a method to compute a Modal Acoustic Transfer Vector (MATV) from an acoustic transfer vector (ATV) for a modal space defined by the Eigen-frequencies of a body, comprising the steps of:
projecting the ATV into the modal space, and
using the MATV to predict a response of the body or the effect of such a response at a reference point remote from the body.

The present invention may also include a processing engine adapted to carry out any of the methods of the present invenrtion.

The present invention also includes a computer program product for executing on a computer, the computer program product executing any of the methods of the present invention. The computer product may be stored on a computer readable data carrier such as one or more diskettes or one or more CD-ROMS or may be transmitted voia a telecommunications network such as the internet.

The present ibvention also includes a method of inputting at a near terminal a representation of a body and coordinates of a reference point and transmitting these to a remote terminal running a program for executing any of the methods of the present invention, and receiving at a near location an output of any of the methods. The output may be one of:

an ATV, a vibrational amplitude such as an acoustic pressure level, a surface vibration of the body, a revised design of at least a part of the body.

The invention consists in one aspect in a new methodology to dramatically speed up the evaluation of the Acoustic Transfer Vectors and their exploitation. In particular the use of the reciprocity principle allows to improve the accuracy of the ATV's because they can be computed and stored on an element basis, and to reduce the computational effort, since the solution needs to be performed for a number of excitation vectors equal to the number of field points of interest, instead of to the number of boundary nodes.

Further on, the use of efficient interpolation techniques is very appropriate for evaluating ATV's, due to their smooth frequency dependence, and allow to restrict their evaluation to a restricted set of master frequency lines, which are then de-coupled from the excitation frequency contents. This results in extreme gains in computing time and data storage space.

The developed methodology results therefore in a reduction of the computational cost attached to acoustic performance prediction, making it quasi insensitive to the number of operational conditions, to the number of frequency lines of interest, and to the number of design variants. As such, it allows the acoustic performance prediction tool to be driven by an automated design exploration and optimization tool.

Benefit of the invention

Acoustic performance prediction is applied to many different kinds of products, in a process that is well standardized. As an example, let us consider an automotive powertrain structure, whose goal is to minimize the acoustic sound emission, and try_to_quantify the benefit due to the invention.

A reasonable boundary element model will include about 7000 nodes/elements. It should be analyzed under 50 different operating conditions, corresponding to different engine speed (RPM's). For building a full frequency response, about 200 frequency lines need to be computed. The performance at 10 microphone locations is of interest, for example to evaluate the radiated sound power (according to the ISO3744-1981 procedure).

The computational cost for processing such a model on an engineering workstation is based on following figures: the matrix assembly and factorization process takes 5,400 seconds per frequency, and the back-substitution process (needed for each excitation case) takes 3 seconds per frequency. Assuming that about 20 different design alternatives need to be analyzed in the design exploration process, we get following estimations:

- Using the traditional acoustic prediction technique, we can estimate the total computational effort to be the number of frequency lines (200) times the time for a single frequency analysis (5,400 sec + 3 sec x 50 load cases), times the number of design alternatives (20), i.e. 22.2 million seconds, corresponding to 257 computing days.

- Using the Acoustic Transfer Vectors technique, computed in a direct way, the model has to be solved for a number of excitation cases equal to the number of nodes (7000), but since the exploitation time (scalar product) is neglectible, the total computational cost is independent on the number of design alternatives, and of operating conditions. Therefore, the total computational effort is estimated to be the number of frequency lines (200) times the time for a single frequency analysis (5,400 sec + 3 sec x 7000 right-hand-side vectors) i.e. 5.28 million seconds, corresponding to 61 computing days.

- Using the invention, i.e. the Acoustic Transfer Vectors technique computed in the reciprocal manner combined with an interpolation technique with 10 master frequencies, and neglecting the time for ATV interpolation and exploitation, we can estimate the total computational effort to be the number of master frequency lines (10) times the time for a single frequency analysis (5,400 sec + 3 sec x 10 load cases), i.e. 54,300 seconds, corresponding to 0.6 computing days.

This example shows that, considering the written assumptions, the invention leads to a reduction of the computational effort of 100 compared to the conventional ATV approach, and 400 compared to the traditional acoustic radiation prediction technique.

THEORETICAL BACKGROUND

Notations

A, B, H, Q	Fluid domain matrix operators
ATV	Acoustic Transfer Vector
ATM	Acoustic Transfer Matrix
c	Sound velocity
C	Coupling matrix
G	Green's function
f	Frequency
k	Wave number
K, D, M	Fluid stiffness, damping and mass matrix operators
j	Imaginary number
n	Surface normal
p	Acoustic pressure
p_b	Boundary acoustic pressure
S	Surface of the vibrating (emitting) structure
U, V	Complex matrices (SVD)
v	Acoustic particle velocity
v_b	Boundary structural velocity
v_s	Structural velocity
x	Observation point
y	Source point
β	acoustic admittance (complex)
ρ	mass density (complex)
σ	Single layer potential; singular values
μ	Double layer potential

ω	Circular frequency
i	Internal degrees of freedom (superscript)
j	Interface degrees of freedom (superscript)

Governing Equation

Acoustic wave propagation in bounded or unbounded regions is governed by the wave equation. Assuming an harmonic behavior, the time and space variables may be separated, leading to Helmholtz' equation:

$$\nabla^2 p + k^2 p = 0$$

where k is the wave number ω/c.

Four numerical formulations are available for solving Helmholtz equation over a region bounded by a surface S, with proper boundary conditions, and to build Acoustic Transfer Vectors in an improved way following the principles of the invention.

1. The direct boundary element method (DBEM) formulation, using the acoustic pressure and velocity as boundary variables. This formulation can be used to model homogeneous interior or exterior domains.
2. The multi-domain direct boundary element method (MDDBEM) formulation, using the acoustic pressure and velocity as boundary variables. This formulation can be used to model a non-homogeneous domain by dividing it in a set of homogeneous sub-domains that are linked together by continuity conditions.
3. The indirect boundary element method (IBEM) formulation, using the single and double layer potentials (acoustic velocity jump and pressure jump respectively) as boundary variables. This formulation can be used to model a homogeneous domain, where both sides of the boundary radiate. In particular, it can be used to model an interior domain, an exterior domain or both simultaneously.
4. The acoustic finite/infinite element method (FEM) formulation, using the acoustic pressure as boundary variables. This formulation can be used to model a non-homogeneous interior domain, an exterior domain or both simultaneously.

Direct Boundary Element Method (DBEM) Formulation

Using the DBEM formulation, the acoustic pressure at any point of a homogeneous fluid domain containing no acoustic source can be expressed in terms of the acoustic pressure on the boundary domain and its normal derivative [1]:

$$p(\vec{x}) = \int_S p(\vec{y}) \frac{\partial G(\vec{x}|\vec{y})}{\partial n_y} dS_y - \int_S \frac{\partial p(\vec{y})}{\partial n_y} G(\vec{x}|\vec{y}) dS_y \tag{1}$$

where $p(\vec{y})$ is the acoustic pressure on the boundary and $\frac{\partial p(\vec{y})}{\partial n_y}$ its normal derivative, $\vec{n}_y$ is the inward normal at point $\vec{y}$ on the boundary and $G(\vec{x}|\vec{y})$ is the Green's function. Making use of the Euler equation, equation (1) becomes:

$$p(\vec{x}) = \int_S p(\vec{y}) \frac{\partial G(\vec{x}|\vec{y})}{\partial n_y} dS_y + j\rho\omega \int_S v(\vec{y}) G(\vec{x}|\vec{y}) dS_y \tag{2}$$

where $v(\vec{y})$ is the normal acoustic velocity on the boundary, ρ is the fluid mass density and ω is the angular frequency. Note that the boundary acoustic velocity is related to the structural velocity through the following relationship:

$$v(\vec{y}) = v_s(\vec{y}) + \beta(\vec{y}) p(\vec{y}) \tag{3}$$

where $v_s(\vec{y})$ is the structural velocity and $\beta(\vec{y})$ the boundary admittance. Using equation (3), equation (2) becomes

$$p(\vec{x}) = \int_S p(\vec{y}) \frac{\partial G(\vec{x}|\vec{y})}{\partial n_y} dS_y + j\rho\omega \int_S \beta(\vec{y}) p(\vec{y}) G(\vec{x}|\vec{y}) dS_y + j\rho\omega \int_S v_s(\vec{y}) G(\vec{x}|\vec{y}) dS_y \tag{4}$$

This equation is true in the domain and on its boundary. Nevertheless, when evaluated on the boundary, the Green's function and its normal derivative become singular. Whereas the last two integrals of equation (4) are regular, the first one is singular and should be evaluated in the Cauchy's principal value sense:

$$c(\vec{x}) p(\vec{x}) = P.V. \int_S p(\vec{y}) \frac{\partial G(\vec{x}|\vec{y})}{\partial n_y} dS_y + j\rho\omega \int_S \beta(\vec{y}) p(\vec{y}) G(\vec{x}|\vec{y}) dS_y + j\rho\omega \int_S v_s(\vec{y}) G(\vec{x}|\vec{y}) dS_y \tag{5}$$

where

$$c(\vec{x}) = 1 + P.V. \int_S \frac{1}{4\pi |\vec{x} - \vec{y}|^2} \frac{\partial |\vec{x} - \vec{y}|}{\partial n_y} dS_y \tag{6}$$

in the three dimensional space. Note that $c(\vec{x})=1/2$ for a smooth surface around $\vec{x}$.

Once discretized using boundary elements and evaluated at the mesh nodes, equation (5) leads to the following matrix system:

$$[A]\{p_b\}=[B]\{v_b\} \tag{7}$$

where the subscript b stands for *boundary*. Similarly, equation (4) gives:

$$p=\{d\}^{\mathrm{T}}\{p_b\}+\{m\}^{\mathrm{T}}\{v_b\} \tag{8}$$

Combining equations (7) and (8) leads to:

$$p=\{atv\}^{\mathrm{T}}\{v_b\} \tag{9}$$

where $\{atv\}$ is the Acoustic Transfer Vector, given by:

$$\{atv\}^{\mathrm{T}}=\{d\}^{\mathrm{T}}[A]^{-1}[B]+\{m\}^{\mathrm{T}} \tag{10}$$

The Acoustic Transfer Vector (ATV) is therefore an array of transfer functions between the surface normal velocity and the pressure at the field point. Finally, when the pressure is evaluated at several locations, equation (9) can be rewritten as:

$$\{p\}=[ATM]^{\mathrm{T}}\{v_b\} \tag{11}$$

where the Acoustic Transfer Matrix $[ATM]$ is formed by the different Acoustic Transfer Vectors.

Multi-Domain Direct Boundary Element Method (MDDBEM) Formulation

In case the acoustic region includes partitions characterized with different fluid material properties, the above formulation can not be directly used since it is only valid for homogeneous domains. In such cases, a multi-domain integral formulation has to be used. The global acoustic region is then decomposed into sub-domains, with the requirement that within a sub-domain, the fluid properties need to be homogeneous. At the interface between the sub-domains, continuity conditions are enforced.

For sake of simplicity, let's consider here a two-domain model. Equation (7) for the first sub-domain can be written as:

$$\begin{bmatrix} A_1^{ii} & A_1^{ij} \\ A_1^{ji} & A_1^{jj} \end{bmatrix}\begin{Bmatrix} p_1^i \\ p_1^j \end{Bmatrix}=\begin{bmatrix} B_1^{ii} & B_1^{ij} \\ B_1^{ji} & B_1^{jj} \end{bmatrix}\begin{Bmatrix} v_1^i \\ v_1^j \end{Bmatrix} \tag{12}$$

where superscript i stands for internal degrees of freedom and superscript j stands for interface degrees of freedom. Similarly, equation (7) can be rewritten for the second sub-domain as:

$$\begin{bmatrix} A_2^{ii} & A_2^{ij} \\ A_2^{ji} & A_2^{jj} \end{bmatrix} \begin{Bmatrix} p_2^i \\ p_2^j \end{Bmatrix} = \begin{bmatrix} B_2^{ii} & B_2^{ij} \\ B_2^{ji} & B_2^{jj} \end{bmatrix} \begin{Bmatrix} v_2^i \\ v_2^j \end{Bmatrix} \tag{13}$$

The continuity of the normal velocity and acoustic pressure has to be satisfied at the interface between of the two sub-domains:

$$\begin{cases} v_1^j = -v_2^j = v^j \\ p_1^j = p_2^j = p^j \end{cases} \tag{14}$$

Combining equations (12), (13) and (14) leads to the global system of equations:

$$\begin{bmatrix} A_1^{ii} & A_1^{ij} & -B_1^{ij} & 0 \\ A_1^{ji} & A_1^{jj} & -B_1^{jj} & 0 \\ 0 & A_2^{ij} & B_2^{ij} & A_2^{ii} \\ 0 & A_2^{jj} & B_2^{jj} & A_2^{ji} \end{bmatrix} \begin{Bmatrix} p_1^i \\ p^j \\ v^j \\ p_2^i \end{Bmatrix} = \begin{bmatrix} B_1^{ii} & 0 \\ B_1^{ji} & 0 \\ 0 & B_2^{ii} \\ 0 & B_2^{ji} \end{bmatrix} \begin{Bmatrix} v_1^i \\ v_2^i \end{Bmatrix} \tag{15}$$

Using equation (15), one can define the Acoustic Transfer Vectors for the global system:

$$p = \{atv\}^T \begin{Bmatrix} v_1^i \\ v_2^i \end{Bmatrix} \tag{16}$$

and similarly derive the Acoustic Transfer Matrix $[ATM]$ when the acoustic pressure has to be evaluated at different locations.

Indirect Boundary Element Method (IBEM) Formulation

Using an indirect formulation, the acoustic pressure at any point of the domain can be expressed in terms of single and double layer potentials [2]:

$$p(\vec{x}) = \int_S \mu(\vec{y}) \frac{\partial G(\vec{x}|\vec{y})}{\partial n_y} dS_y - \int_S \sigma(\vec{y}) G(\vec{x}|\vec{y}) dS_y \tag{17}$$

where $\sigma(\vec{y})$ and $\mu(\vec{y})$ are the single and double layer potentials, respectively. For the Neuman problem (i.e. structural velocity boundary condition) and when both sides of the boundary have the same velocity (i.e. thin shell), this equation can be expressed only in terms of double layer potentials:

$$p(\bar{x}) = \int_S \mu(\bar{y}) \frac{\partial G(\bar{x}|\bar{y})}{\partial n_y} dS_y \tag{18}$$

Equation (18) can be evaluated on an arbitrary surface S'. It can be derived with respect to the normal at $x \in S'$, pre-multiplied by an admissible test function $\delta\mu(x)$ and integrated over the surface S':

$$\int_{S'} \frac{\partial p(\bar{x})}{\partial n_x} \delta\mu(\bar{x}) dS' = \int_{S'} \int_S \mu(\bar{y}) \frac{\partial^2 G(\bar{x}|\bar{y})}{\partial n_x \partial n_y} \delta\mu(\bar{x}) dS_y dS'_x \tag{19}$$

Now, let the surface S' tend to S and make use of the Euler equation:

$$-\int_S j\omega\rho v(\bar{x}) \delta\mu(\bar{x}) dS = \int_S \int_S \mu(\bar{y}) \frac{\partial^2 G(\bar{x}|\bar{y})}{\partial n_x \partial n_y} \delta\mu(\bar{x}) dS_y dS_x \tag{20}$$

After discretization using boundary elements equation (18) becomes:

$$p = \{d\}^{\mathrm{T}} \{\mu\} \tag{21}$$

Similarly, equation (20) becomes:

$$[Q]\{\mu\} = [H]\{v_b\} \tag{22}$$

Combining equations (21) and (22), we obtain:

$$p = \{atv\}^{\mathrm{T}} \{v_b\} \tag{23}$$

where $\{atv\}$ is the Acoustic Transfer Vector, given by:

$$\{atv\}^{\mathrm{T}} = \{d\}^{\mathrm{T}} [Q]^{-1} [H] \tag{24}$$

When the acoustic pressure is evaluated at several locations, equation (23) can be rewritten as:

$$\{p\} = [ATM]^{\mathrm{T}} \{v_b\} \tag{25}$$

where the Acoustic Transfer Matrix $[ATM]$ is formed by the different Acoustic Transfer Vectors.

Finite/infinite element method (FEM/IFEM) formulation

The solution of Helmholtz' equation based on a finite/infinite element approach leads to a system of linear equations:

$$([K]+j\rho\omega[D]-\omega^2[M])\{p\}=-j\rho\omega\{F\} \tag{26}$$

where K is the fluid stiffness matrix, D is the fluid damping matrix, M is the fluid mass matrix and F is the forcing vector, defined by:

$$\{F\}=[C]\{v_b\} \tag{27}$$

where C is a coupling matrix.

Combining equations (26) and (27).), we obtain:

$$\{p\}=[ATM]^T\{v_b\} \tag{28}$$

where the Acoustic Transfer Matrix $[ATM]$ is given by:

$$[ATM]^T=\frac{-j\rho\omega[C]}{([K]+j\rho\omega[D]-\omega^2[M])} \tag{29}$$

Finally, for each finite element grid point, the acoustic pressure can be written as:

$$p=\{atv\}^T\{v_b\} \tag{30}$$

where $\{atv\}$ is the Acoustic Transfer Vector, given by the corresponding row of the $[ATM]^T$ matrix.

Use of the Reciprocity Principle

A key element in the described invention is the use of the reciprocity principle for efficiently computing the Acoustic Transfer Vectors. As shown in equations (10), (15), (24) and (29), the evaluation of any field point related ATV requires the inversion of a matrix, whose order is approximately equal to the number of nodes of the discretized geometry (mesh).

The numerical evaluation of the inverse of an order n matrix is performed by solving a system of linear equations, whose right-hand-side is built from the unity matrix, leading typically to one matrix factorization, and n back-substitution steps.

$$[A][X]=[I] \tag{31}$$

For large matrix orders, the computational effort and time required for performing the n back-substitution steps is very significantly larger than the factorization time.

Looking to equation (10), we can deduct that an ATV is a vector for which each coordinate corresponds to the pressure at the corresponding field point due to a unit excitation (vibrating velocity) at one point/element on the surface, and no excitation elsewhere (see figure 4).

$$atv_i = \{atv\}\begin{Bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{Bmatrix} \tag{32}$$

According to the reciprocity principle, the pressure response at a first location due to an excitation at a second location is strictly equal to the pressure response at the second location due to the same excitation at the first location. Therefore, if a source of strength Q_1 at position 1 causes a pressure p_2 at position 2, and if a source of strength Q_2 at position 2 causes a pressure p_1 at position 1, we have the following relationship:

$$p_1 Q_1 = p_2 Q_2 \quad or \quad \frac{p_1}{Q_2} = \frac{p_2}{Q_1} \tag{33}$$

Using the reciprocity principle to evaluate the ATV related to a specific field point means that a monopole source (point source) of strength Q_1 should be positioned at the field point location 1, and the pressure is calculated on the boundary p_2 (see figure 5).

Equation 11 explicates how the ATV will be exploited to convert the boundary surface velocity (representing vibrations) into the field point acoustic pressure. Since a structure may be complex, containing T-junctions and other discontinuities, surface vibrations are better defined on surfaces, i.e. on finite elements, rather than at grid points (where the normal directly is not always uniquely defined). Therefore, it is required to define element-based acoustic transfer vectors.

In the case where the source placed on the boundary (in 2) is distributed over a small surface, equation (33) may be rewritten as:

$$p_1 Q_1 = \int_S p_2 v_2 dS \tag{34}$$

where v_2 is the source velocity. In the particular case where the source is distributed over a single element, the pressure at any point on the surface can be approximated as:

$$p_2 = \langle p_2 \rangle \{N\} \tag{35}$$

where $\{N\}$ is the interpolation function and $\langle p_2 \rangle$ is the pressure evaluated at the element nodes. Similarly the source velocity can be approximated as:

$$v_2 = \langle N \rangle \{v_2\} \tag{36}$$

where $\{v_2\}$ is the source velocity at the element nodes. Equations (34) to (36) lead to the following relation:

$$p_1 Q_1 = \langle p_2 \rangle [C_e] \{v_2\} \tag{37}$$

where $[C_e]$ is the element coupling matrix defined as:

$$[C_e] = \int_{S_e} \{N\}\langle N \rangle dS \tag{38}$$

For a monopole source, the source strength is given by:

$$Q_1 = \frac{4\pi}{j\rho\omega} A \tag{39}$$

where A is the monopole source amplitude. Combining (37) and (39):

$$p_1 = \frac{j\rho\omega}{4\pi A} \langle p_2 \rangle [C_e] \{v_2\} \tag{40}$$

which can be rewritten as:

$$p_1 = [ATV_e] \{v_2\} \tag{41}$$

where $[ATV_e]$ is the element-based acoustic transfer vector, computed by:

$$[ATV_e] = \frac{j\rho\omega}{4\pi A} \langle p_2 \rangle [C_e] \tag{42}$$

Applying at the field point a monopole source with a unit amplitude ($A=1$ and $\omega=2\pi f$, we finally obtain the following expression for evaluating the element-based acoustic transfer vector:

$$[ATV_e] = \frac{j\rho f}{2} \langle p_2 \rangle [C_e] \tag{43}$$

showing that the ATV related to a given field point can then be computed from the boundary sound pressure values, correctly scaled.

The use of the reciprocity principle for evaluating the ATV's leads to two fundamental advantages:

1. The ATV's can be directly computed on an element basis, removing the ambiguity inherent to nodal values, in case of sharp edges and T-junctions, where the normal vector is not uniquely defined.

2. The ATV related to a given field point can then be computed with a single excitation vector, representing the monopole source excitation at the field point. This leads to a single back substitution step. Since the number of field points (microphones locations) is usually several orders of magnitude lower than the number of boundary nodes, this results in tremendous gains in computational effort.

Interpolation technique

In case of sound wave propagation in an open space, the fluid domain does exhibit any resonant behavior. The sound field is fully determined by various wave mechanisms, such as reflection by solid objects, absorption on damping surfaces, diffraction around edge and wave interference. Therefore, Acoustic Transfer Vectors are rather smooth functions of the frequency, and ATV coefficients can be accurately evaluated at any intermediate frequency, called *slave* frequency, using a mathematical interpolation scheme, based on a discrete number of frequencies, called *master* frequencies. It is important to note that the structural vibrations cannot be similarly interpolated, since these are directly dependant on the highly resonant dynamic behavior of any structure (vibration mode shapes).

Two interpolation mechanisms are used:

1. A linear interpolation scheme (see figure 6);
2. A cubic spline interpolation scheme (see figure 7), which is particularly well suited for approximating ATV's.

Other interpolation schemes can be used.

The use of an interpolation scheme for evaluating the ATV's leads to following fundamental advantages:

1. It allows to build a complete frequency dependant ATV, with a very limited number of frequency lines to be computed. In order to build a complete frequency response with a traditional approach, the number of frequency lines to be computed is usually over a hundred. Using an interpolation mechanism, only a few master frequency lines needs to be explicitly computed, any other frequency being interpolated. The gain in computing time is usually over an order of magnitude.

2. Similarly, the ATV's have to be stored in a file database in order to be further exploited. The interpolation mechanism allows to store only the master frequency ATV's in the database. The corresponding vector at any intermediate frequency may then be directly evaluated at a neglectible cost, and therefore does not need to be stored in the database. The gain in disk space is usually over an order of magnitude.

3. For some specific applications, such as automotive powertrains, the acoustic performance has to be evaluated for many different operational conditions (e.g. different rpm's), each of these having a different frequency contents. Using the ATV interpolation mechanism, the same set of master frequencies can be used to build a unique basis of ATV's, which are then interpolated and combined with the structural response corresponding to any load case, at any intermediate frequency needed for the specific operational condition.

Inverse Numerical Acoustics

Equations (11) and (25) are the basic relations for the inverse numerical acoustics theory. The sound pressure is measured at a large number of microphone (field points) and the boundary velocity can be obtained using the relationship:

$$\{v_b\} = [ATM]^{-1}\{p\} \tag{44}$$

However, the inversion of the acoustic transfer matrix is not obvious, because the matrix is generally not square but rectangular and because it involves a Fredholm equation (of the first kind for the indirect approach and of the second kind for the direct approach) and is ill-conditioned. Therefore the matrix inversion is performed using the singular value decomposition (SVD) technique, which allows to solve singular or close to singular systems. It is based on the fact that any $n \times m$ complex matrix can be written as:

$$[ATM] = [V][\sigma][U]^{H} \tag{45}$$

where the superscript H stands for 'transpose complex conjugate', $[\sigma]$ is a diagonal $\min(n,m) \times \min(n,m)$ real matrix, $[U]$ is a $m \times \min(n,m)$ complex matrix and $[V]$ is a $n \times \min(n,m)$ complex matrix. The coefficients of $[\sigma]$, called singular values, are stored in an decreasing order and matrices $[U]$ and $[V]$ are such that:

$$[V]^{H}[V] = [U]^{H}[U] = [I] \tag{46}$$

From equations (45) and (46) it follows:

$$[ATM]^{-1} = [U][\sigma]^{-1}[\mathrm{V}]^{H} \tag{47}$$

Nevertheless, singular or ill-conditioned matrices contain null or small singular values. This results in infinite or very large values in $[\sigma]^{-1}$. Whereas the infinite terms clearly lead to an infinite solution, the very large values lead to a solution of equation (44) that will be very sensitive to the right hand side variations. Therefore, small errors on the RHS will result in very large errors in the solution. To avoid this problem, the small or null terms of $[\sigma]$ are set to zero in $[\sigma]^{-1}$. This process is called truncated singular value decomposition. The singular values are set to zero as soon as $\sigma_i < \alpha\sigma_1$ where α is a tolerance parameter.

Panel Contribution Analysis

Panel Contribution Analysis (see figure 8) is a fundamental engineering tool to guide the product refinement in the development process. Equations (9), (16), (23) or (30) can be rewritten as:

$$p = \sum_{e=1}^{ne} p_e = \sum_{e=1}^{ne} [ATV_e]^T \{v_e\} \qquad (48)$$

where ne is the total number of elements. Subsequently, the contribution of a panel is given by the summation of element contributions over the panel elements:

$$p_c = \sum_{pe} p_e = \sum_{pe} [ATV_e]^T \{v_e\} \qquad (49)$$

where pe stands for panel elements.

Modal ATV

The engineering process to compute the structural velocity $\{v_b\}$ on a vibrating surface relies usually on the structural finite element method, and often on a modal superposition approach, where the structural response is expressed as a linear combination of the mode shapes of the body:

$$\{v_b\} = [\Phi_n]\{mpf\} \qquad (50)$$

Where $[\Phi_n]$ is the matrix composed of the modal vectors, projected on the normal direction to the boundary surface, and $\{mpf\}$ are the modal participation factors, also called the modal response of the structural model, at a given excitation frequency.

Combining (50) with Equations (9), (16), (23) or (30) leads to:

$$p = \{atv\}^T [\Phi_n]\{mpf\} \qquad (51)$$

where

$$\{atv\}^T [\Phi_n] = \{matv\}^T \qquad (52)$$

is called the Modal Acoustic Transfer Vector, which can be directly combined with the modal response to give the sound pressure at a field point:

$$p = \{matv\}^T \{mpf\} \qquad (53)$$

The concept of Modal Acoustic Transfer Vectors can be extended to any possible deformation shapes of the body, so-called Ritz vectors. This is especially interesting in the case of inverse numerical acoustics, where the unknown becomes then the modal participation factors.

Optimization strategy

A method has been developed that allows the numerical acoustic radiation prediction calculation to be extremely fast, the main computational intensive task (calculation of the ATV's) being done as a pre-processing step. Therefore, it is directly possible and practical to integrate the acoustic radiation prediction within an optimization loop, where the objective function is the acoustic performance (sound pressure levels, or radiated power according to the ISO3744-1981 procedure), and where the design variables are either structural or acoustic design variables.

The optimization process is shown at figure 9. It will typically combine a structural dynamic finite element solver together with an ATV-based (or MATV) acoustic prediction tool.

Implementation

The methods of the present invention may be implemented on a processing engine such as a workstation or a personal computer. The processing engine may be a server accessible via a telecommunications network such as a LAN, a WAN, the Internet, an Intranet. The server may be adapted to carry out any of the methods of the present invention, for example a descriptor file of a body may be entered at a near terminal an transmitted to the server via the Internet. The server then carries out one of the methods of the present invention and returns to a near location e.g. an e-mail box, any response of any of the methods in accordance with the present invention. Such a response may be, for instance, an ATV, a vibrational amplitude such as an acoustic pressure level, a surface vibration of the body, a revised design of at least a part of the body.

The methods of the present invention may be implemented as computer programs which may be stored on data carriers such as diskettes or CD-ROMS. These programs may also be downloaded via the Internet or any other telecommunications network.

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CLAIMS

1. A method for computing a Wave Transfer Vector based on the reciprocity principle; comprising the steps of:
   simulating positioning of a monopole, omnidirectional wave energy source at a reference position remote from a body;
   computing a boundary oscillation amplitude of the wave generated by the source at a surface of the body;
   deriving from the boundary oscillation amplitude said Wave Transfer Vector.

2. The method of claim 1 wherein the computing step is carried out by a numerical method.

3. The method according to claim 2 wherein the numerical method is the finite element method.

4. The method of claim 2 wherein the numerical method is a combination of the finite and infinite element methods.

5. The method of claim 2 wherein the numerical method is the direct boundary element method.

6. The method of claim 2 wherein the numerical method is the direct multi-domain boundary element method.

7. The method of claim 2 wherein the numerical method is the indirect boundary element method.

8. The method according to any previous claim, wherein wave source is an acoustic source.

9. A method for computing an additional Wave Transfer Vector comprising the steps of:
   computing at least a first and a second wave transfer vector at a first and a second predetermined frequency, respectively, and
   computing the additional Wave Transfer Vector at a frequency intermediate the first and second frequency by interpolation between the first and second Wave Transfer Vectors.

10. The method of claim 9 wherein the interpolation technique is the linear interpolation mechanism.

11. The method of claim 9 wherein the interpolation technique is the cubic spline interpolation mechanism.

12. A method to compute a Modal Acoustic Transfer Vector (MATV) from an acoustic transfer vector (ATV) for a modal space defined by the eigen frequencies of a body, comprising the steps of:
projecting the ATV into the modal space, and
using the MATV to predict a response of the body or the effect of such a response at a reference point remote from the body.

13. The combination of methods of claims 1 and 9 for supporting the acoustic radiation prediction engineering process, in particular but not limited to automotive engine and powertrain noise radiation application.

14. The combination of methods of claims 1, 9 and 12 for supporting the modal-based acoustic radiation prediction engineering process, in particular but not limited to automotive engine and powertrain noise radiation application.

15. The combination of methods of claims 1 and 9 for supporting the inverse numerical acoustics engineering process, in particular but not limited to automotive engine and powertrain noise radiation application.

16. The combination of methods of claims 1, 9 and 12 for supporting the modal-based inverse numerical acoustics engineering process, in particular but not limited to automotive engine and powertrain noise radiation application.

17. The method of claim 12, wherein shapes combined to build a structurall response are Ritz Vectors.

18. The combination of methods of claims 1, 9 and 17 for supporting the Ritz-vectors-based inverse numerical acoustics engineering process, in particular but not limited to automotive engine and powertrain noise radiation application.

19. The combination of methods of claims 1 and 9 for supporting the evaluation of multiple design alternatives, with respect to the acoustic performance prediction of vibrating structures.

20. The combination of methods of claims 1, 9 and 12 for supporting the evaluation of multiple design alternatives, with respect to the modal-based acoustic performance prediction of vibrating structures.

21. The combination of methods of claims 1 and 9 for supporting an automated structural-acoustic optimization process, with the objective function being the acoustic performance prediction of vibrating structures.

22. The combination of methods of claims 1, 9 and 12 for supporting an automated structural-acoustic optimization process, with the objective function being the modal-based acoustic performance prediction of vibrating structures.

23. A processing engine adapted to carry out any of the methods of claim 1 to 22.

24. A computer program product for executing on a computer, the computer program product executing any of the method steps of claim 1 to 22 when executed on the computer.

25. A method of inputting at a near terminal a representation of a body and coordinates of a reference point and transmitting these to a remote terminal running a program for executing any of the methods of claim 1 to 22, and receiving at a near location an output of any of the methods.

26. The method according to claim 25, wherein the output is one of:

an ATV, an oscillation amplitude such as an acoustic pressure level, a surface vibration of the body, a revised design of at least a part of the body.

ABSTRACT

The invention relates to Computer-Aided Engineering (CAE) systems. It concerns a new methodology to predict (1) the acoustic radiation characteristics of a mechanical structure, under operational conditions, and (2) to identify the sources on a vibrating structure from measured sound pressure levels in the field. The methodology is based on a new approach to evaluate acoustic transfer vectors, based on the reciprocity principle and combined with interpolation techniques.

The same methods are applicable to other vibrating energy forms which can be described by the wave equation such as electromagnetic waves.

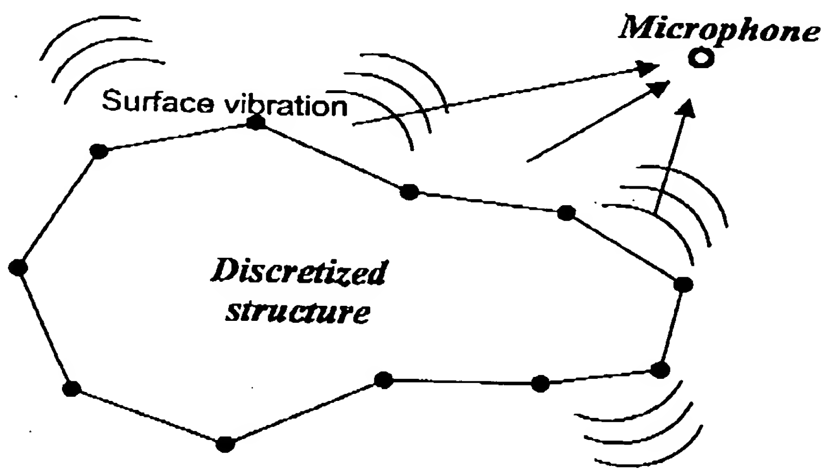

Figure 1: Acoustic Radiation Prediction.

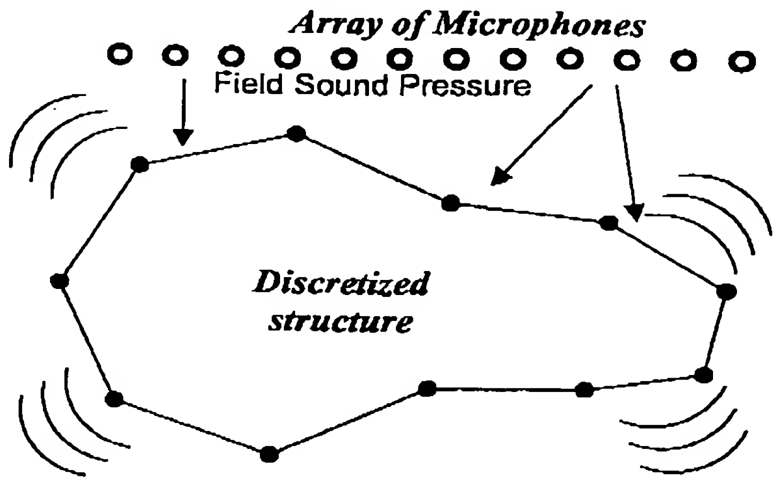

Figure 2: Inverse Numerical Acoustics.

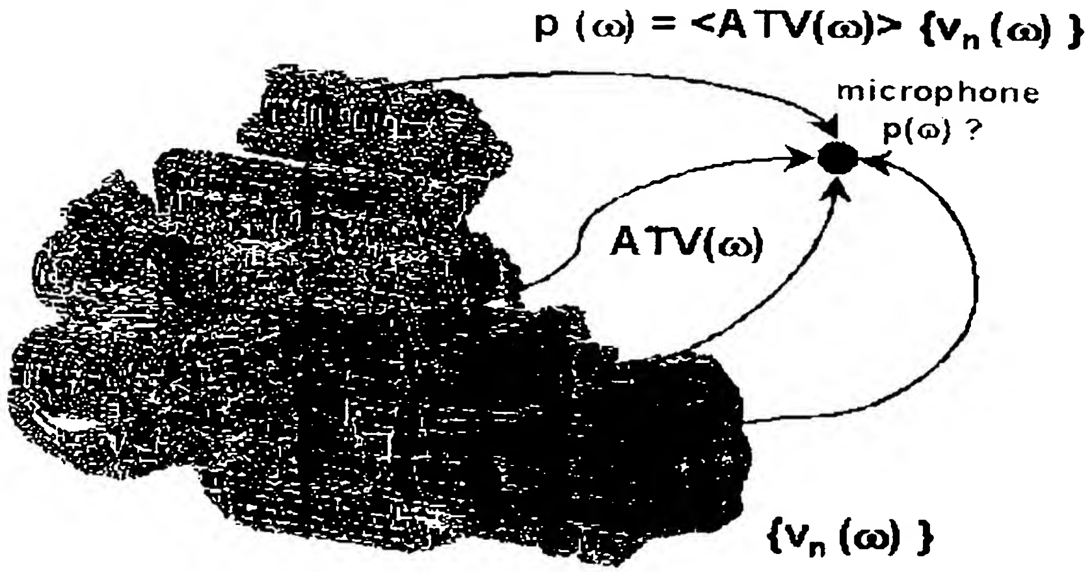

Figure 3: The ATV-concept

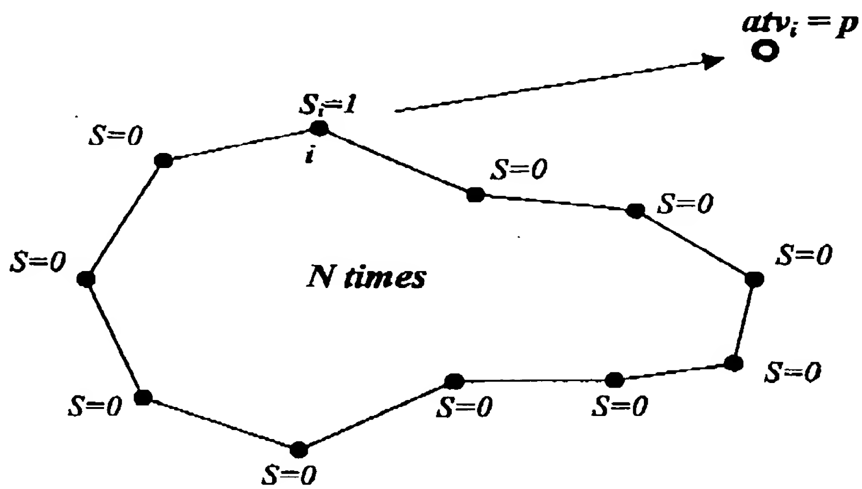

Figure 4: Direct method for evaluating ATV's. This has to be repeated n times.

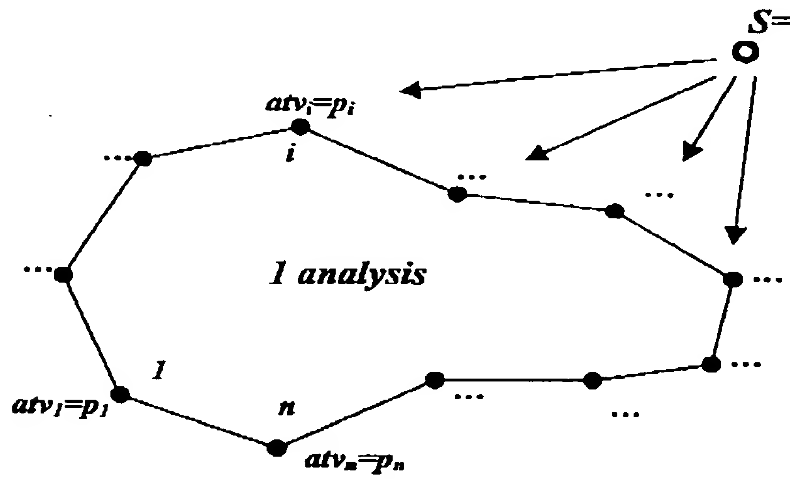

Figure 5: Reciprocal method for evaluating ATV's.

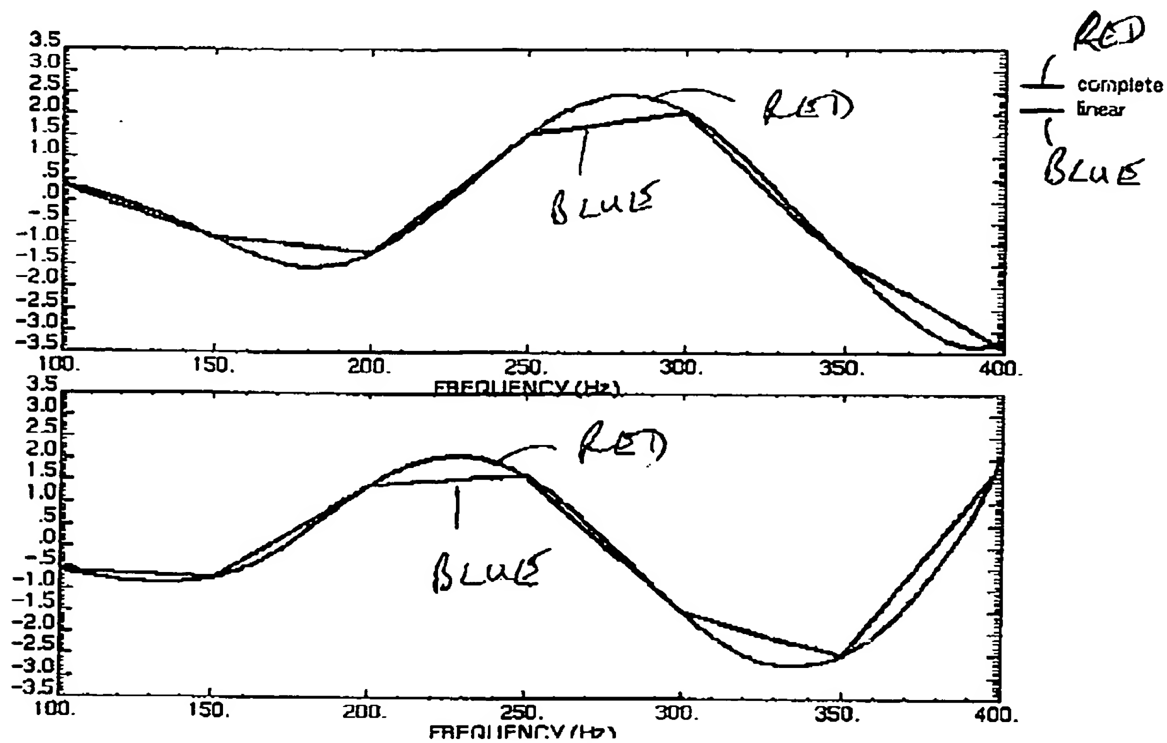

Figure 6: linear interpolation (in blue) of an Acoustic Transfer Vector (in red).

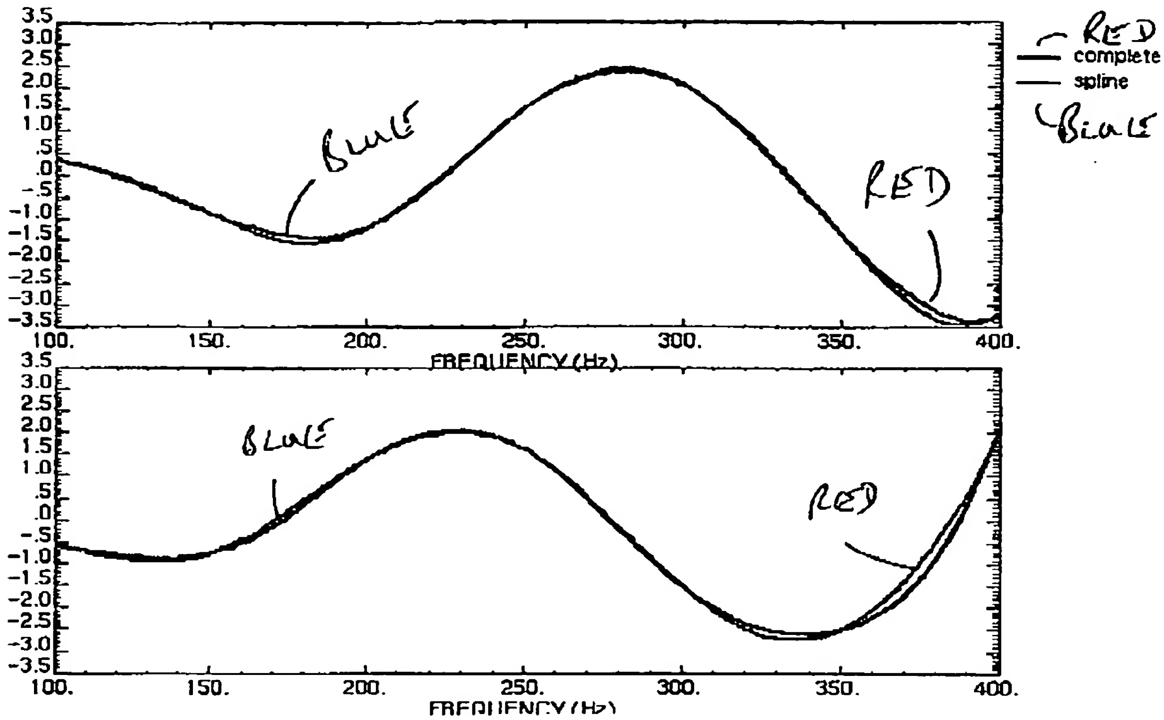

Figure 7: Spline interpolation (in blue) of an Acoustic Transfer Vector (in red).

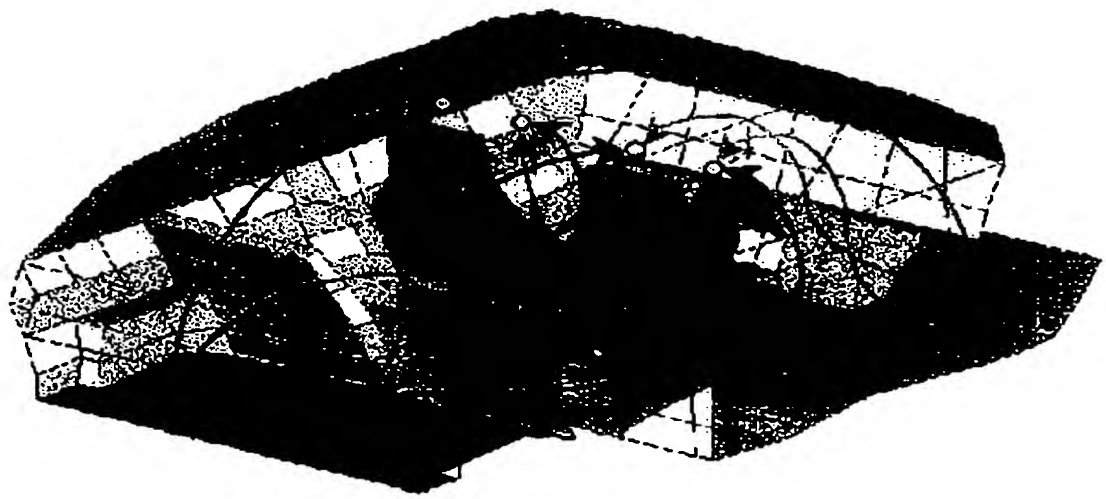

Figure 8: Panel Acoustic Contribution Analysis using a MDDBEM model

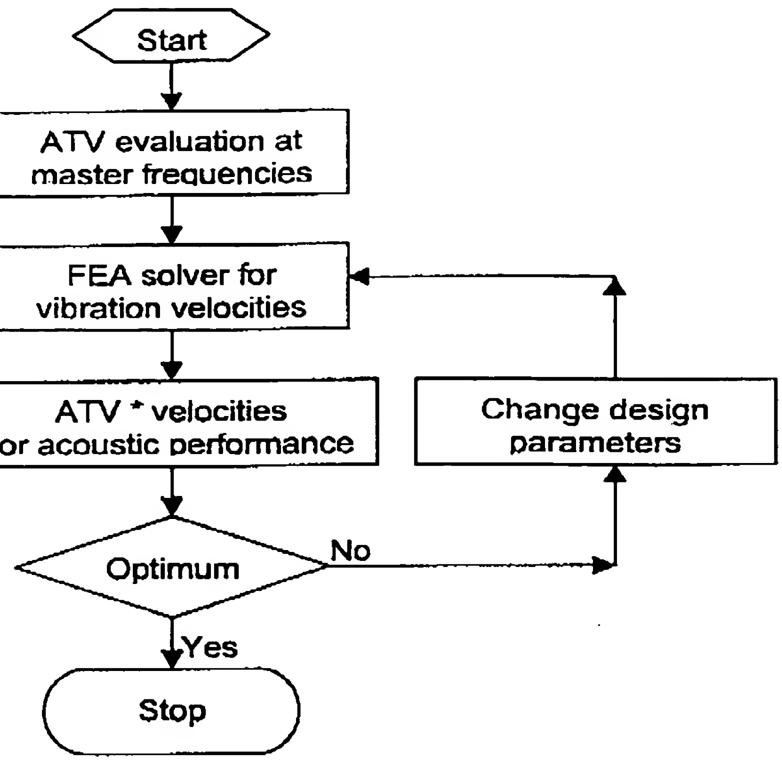

Figure 9: Optimization process based on ATV's.